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What are the Objects of *Dianoia*?

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ABSTRACT

In this paper, I examine the problem of the so-called Mathematical Objects within the context of the Divided Line. I argue that Plato believes that there are such objects but their distinctness and the mode of cognition relative to them can only be understood in relation to the superordinate, unhypothetical first principle of all, the Idea of the Good. The objects of mathematics or διάνοια are, unlike the objects of intellection or νόησις, cognized independently of the Good.

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Aristotle's testimony that Plato introduced "mathematicals" (τὰ μαθηματικά) in between (μεταζύ) Forms (τὰ εἴδη) and sensibles (τὰ αἰσθητά) raises a host of problems, the various solutions to which are no doubt familiar to everyone here.[1] I suppose that any effort to move the discussion forward has to start with some indisputable claims made by Plato especially in the Divided Line passage. I shall begin simply by listing the claims that seem to me to be indisputable or nearly so and then move on quickly to highly disputed territory. Here are the claims that in my opinion should shape our reflections on the putative status of mathematicals:

1. In the Divided Line, Plato makes a hard distinction between the intelligible world and the sensible world, between that which is available only to thought and that which, though available to thought, is so only on the basis of sense-perception.[2]
2. There are a number of correlations or improper proportionalities in the Divided Line: just as (A) the mode of cognition for the top section (ἐπιστήμη) is to (B) the mode of cognition for the bottom section (δόξα), so in the top section, (A1) the mode of cognition in the first part of the top section (νόησις) is to (A2) the mode of cognition in the second part (διάνοια), as in the bottom section, (B1) the mode of cognition of the first part (πίστις) is to (B2) the mode of cognition of the second part (εἰκασία).[3]
3. The distinction between each of the objects of the four modes of cognition is as irreducible as are the distinctions among the four modes of cognition themselves. The distinctions among the objects are to be made according to the criteria of clarity (σαφηνεία) and truth (ἀλήθεια). The first of these criteria suggests the possibility of continuous gradation; on first glance the second does not. Nevertheless, if truth here is "ontological" truth and not "semantic truth," then gradation may well parallel that of clarity. I understand ontological truth as a relational property of Being, the property of being intelligible or transparent to our intellects. So, objects of cognition can be more or less clear and more or less true, that is, more or less intelligible to us. That means that there is a gradation of intelligiblity according to the criterion, roughly, of transparency to an intellect alone. The sensible world is less intelligible than the invisible world because the former is enmeshed in that which is unintelligible or opaque to an intellect (as opposed to the senses).

According to the description of διάνοια and its objects along with Aristotle's testimony, we might conclude that διάνοια has as its objects mathematicals, whereas νόησις has as its objects the paradigms of these: instead of multiple triangles, Triangularity; instead of multiple numbers, Form-Numbers.[4] When the mathematician says, "let there be a triangle," or "take a prime number," using sensible images of these, he is striving to achieve cognitive success regarding Triangularity or Number. Since such success does not turn διάνοια into νόησις, we need to ask first why this is so.

At A1, in order to have νόησις, one must "ascend" to the Good and then "descend" through Forms, presumably back to the Form we started with. Why, if the mathematician

provides a proof of a theorem in geometry, is this not enough? That is, why should the ascent and descent be so significant that it leads Plato to distinguish two modes of cognition, one not making the ascent and descent and one making it?[5] Why is there more clarity and truth in νόησις than in διάνοια?

In order to be able to answer these questions, let us make concrete the mathematical posits. Assume there is a Form of Three and a Form of Four. Supposedly, the Idea of the Good somehow makes these knowable. Since the Forms are not composed of units, they are not addable. It is not the case that the Form of Three and the Form of Four when added are the Form of Seven. Nevertheless, when the mathematician adds the numbers three and four, he gets seven. And it is something about the Form of Three and the Form of Four and the Form of Seven that guarantees that this is an eternal and necessary truth. Stated otherwise, what is needed is a metaphysical explanation of mathematical necessity. This explanation would tell us why a mathematician when doing mathematics does not invent mathematical truths but discovers them.

So, the Form of Three and the Form of Four are to be eternally related such that three plus four is seven is an eternal necessary truth. According to the text, it is the Idea of the Good that enables us to see this relation, and appreciate that it is other than the relation of addition, given that Form-Numbers are not addable. It is, I think, nonsense to maintain that it is the goodness of the Idea of the Good that enables us to see this relation, that once we see that there is an Idea of the Good, we can then see that Three and Four are eternally related such that they guarantee that when we add three and four we get seven. It may well be that in some sense it is good that this is so. But it is unintelligible (to me anyway) that knowing that this is so is necessary for knowing the Forms for one very good reason. If "good" is a property of a Form, we cannot know that the Form is good unless and until we know what that Form is. This is of course the principle laid down in *Euthyphro*.[6] So, if we have to "ascend" to the Good in order to know what a Form is, the claim being made is not that we need to be able to see that the Form is good or, stated otherwise, that it is good that the Form be what it is. It is not the predicative nature of goodness that is at issue, so to speak, but the metaphysical principle, the Good.

The opacity of the claim that the Good is the source of the knowability of Forms and that, in addition, the Forms cannot be known without "ascending" to the Good evidently defeats the ingenuity of those scholars who try to understand what is going on here while staying within the confines of *Republic*.[7] By contrast, the entire Platonic tradition up to the beginning of the 19th century was in accord in taking as evident that we needed to have recourse to Aristotle's testimony about Plato's philosophy and the testimony of the indirect tradition in order to make any progress.[8] Aristotle and the indirect tradition of Platonism do not suppose that it is the Good as such that explains the relatedness of Three and Four; rather, it is because the Good is the One that we can see how the first principle of all unifies the Forms and thus explains the eternal relatedness of Three and Four and *all* the rest of the Forms.

The One is certainly not the number one. Nor is it a unit. The most perspicuous way of describing it is as a uniquely absolutely simple explanatory principle. That is, it is unqualifiedly incomposite. There can be no more than one unqualifiedly incomposite "things"; in addition, there can be no less than one unqualifiedly incomposite "thing". This is evident by inspection (though also derivable from the second hypothesis of the second part of *Par-*

menides). If there were, counterfactually, two absolutely simple first principles, they would have to differ in at least one property. But if either of the relata has a property, it is not simple. So, there cannot be more than one. There must be at least one absolutely simple explanatory principle because without this, we could never arrive at an adequate or sufficient explanation for anything (the τι ἱκανόν) of *Phaedo*. We would, *ex hypothesi,* be left with something complex, something whose very complexity is left unexplained. The relevant complexity at issue for Plato is that of the existence of something with a specific nature. The complexity is owing to the fact that there is a real distinction between the existence and the essence of such a thing. The existence of something or other is not self-explaining. Knowing what, say, F-ness is, does not explain the existence of something whose nature is F-ness. A clear example of this point is found in *Philebus* (15B) where the question of the existence of "monads" like Man, Ox, Beautiful, Good is raised. Implicit in this question is a distinction between *what* the object of the question is and *whether or not* it exists.

How, then, does the Good-as-One give us the answer to our questions? In fact, for a multitude of reasons, the use of the term "thing" for the One is a mistake. Not the least of these is that if the One is a thing, then it can have a real relation to other "things". But if this were so, then the One as relatum would be really distinct from the property that it has in relation to other things. But an absolutely simple first principle cannot be complex in any way. So, while things may be really related to it, it cannot be really related to these things. This logical point will be crucial to understanding the central role of the first principle of all in all types of cognition, including διάνοια.

The One is the principle of everything because it is virtually everything, analogous to the way that white light is virtually all the colors of the spectrum or the way in which a function is virtually its domain and range. It is one thing to grasp a function in its universality. It is quite another thing to grasp a manifestation or application of that function *as such*. And it is yet another thing to grasp the manifestation without knowing that it is a manifestation of that function. This last cognitive achievement is what children do in basic arithmetic before they learn algebra. It is also, by the way, what computers do when they apply the function as a rule without grasping the function in its universality.

The One is virtually all Being not just in the above sense, but in its absolute simplicity. What this means, roughly, is that all Being is virtually one in the principle of Being just as blue and yellow are virtually one in their principle, white light, or $9 = 3^2$ is virtually $x = y^2$ in its principle. So, all the Forms are virtually one. But they are virtually one *only* in their principle. This means that Three and Four are not really or essentially one; they are distinct Forms. They are distinct for an intellect, which is able to see also that they are virtually one. Since the One is uniquely simple, it follows that everything else is complex including, each Form. Yes, the Form is a one-over-many, but this relative unity is not absolute oneness, as we learn, again, from the second hypothesis of the second part of *Parmenides*.

So, when you think that three plus four is seven, you can either not see that the Form of Three and the Form of Four are virtually identical with the Form of Seven or you can see that this is so. If you don't see that this is so, you are only doing mathematics. If you *do* see that this is so, you are doing dialectic or philosophy. That is, you can see that the Forms of Three and Four are virtually identical with the Form of Seven in their principle while being

distinct in their Being. You can see that Being is a one-many because the principle of Being is unqualifiedly or absolutely One. The Form of Living Animal in *Timaeus* (30C-D) which "contains" all the Forms of living things, is virtually all of these. This means, among other things, that all the Forms of living things are internally related analogous to the way that all the integers are internally related. Five cannot be what it is if it is not internally related to Four and Six. But since Forms are not addable, the internal relatedness is not expressible as one Form having one more or one less unit. Rather, the internal relatedness is, again, as the second part of *Parmenides* suggests, that of ordinality or succession.[9] A somewhat more complex internal relatedness is found in the Periodic Table of the elements.

Before we explore further what this means for the Divided Line and for cognition generally, it will be useful to consider the unintelligibility of any alternative explanation that does not adduce the One. Thus, without the principle that unifies the Forms, the Form of Three and the Form of Four (while not addable and while explaining the addability of three and four) are eternally and necessarily related. That is, they are internally related. But the idea of an immaterial internal relation is opaque, to say the least, unless that relation is the relation of identity. Of course, one will object that if it were as simple as that — that is, if Three and Four were identical — then so would Three and Five, and when we add three and four, we could just as well say eight as we could say seven. But the identity relation is in fact that of *relative* identity not formal identity. Only the One is unqualifiedly self-identical; everything else is composite and so everything else has the identity of a composite, which is part of what I mean by relative identity. So, the relative self-identity of Three and the relative self-identity of Four are to be understood as expressions of the relative self-identity of Seven and the relative identity of Three and Four, on the one hand, and Seven, on the other, are expressions of Being. What it means for Being to be relatively self-identical is, among many other things, that one expression of that is the relative identity of three and four with seven and another is the relative identity of three and five with eight. The claim that the necessary truths of mathematics are expressions of Being is not vacuous because Being is not the first principle of all, but the internal complexity of Being requires that we postulate such a principle. This is hardly vacuous.

The term "expression" which I have just introduced takes us closer to understanding διάνοια and its objects. The term is intentionally ambiguous between (a) a manifestation of Being; (b) a proposition used to represent (a); and (c) the assertion by someone of (b). Incidentally, supposing that ἐπιστήμη for Plato has expressions of type (b) as objects is clearly mistaken. The objects of ἐπιστήμη, as the central books of *Republic* tells us, are Forms and expressions of type (b) by means of (a) are, variously, expressions of this ἐπιστήμη, which is a πάθος in the soul not a relationship — intentional or otherwise — between a soul and a proposition.

The proposition 3 + 4 = 7 is an expression of type (b); the asserting of that proposition is an example of an expression of type (c). The internal relations among Forms logically requires their reductive unity by a first principle which stands above or over Being. The "equal-sign" in the proposition indicates the reductive unity of (a). That is, the numbers 3 and 4, on the one hand, and 7, on the other, are two ways of manifesting or expressing Being, that is, type (a). And a proposition (b) asserted by someone in an expression of type (c) states, roughly, that

two "things" are in reality (read: as they are found in Being) actually one.

The idea of thinking that two (or more) things are in reality one is familiar to us in the Fregean example in which the Morning Star is the Evening Star. Long before Frege, however, Aristotle distinguished between there being two λόγοι, but one "thing" in reality.[10] Thus, teaching and learning are different in λόγος but one in εἶναι. Indeed, Aristotle's entire theory of demonstrative knowledge supposes that species and genera of individual substances can be shown to be really relatively identical with their commensurately universal properties by means of the so-called middle term, the definition of the subject. Thus, that to which "human being" refers and that to which "risible" refers are identical, even though the λόγος of human being and the λόγος of risible are distinct. More generally, our ability to identify anything entails our ability to re-identify it. This is so simply because the ability is not time-bound as is that which we identify. Your ability to identify a smell is the identical ability used when re-identifying it. But that which we identify and then re-identify is one thing expressed as two (or more). So, when I say that S at t_1 is identical with S at t_2, I am asserting that one "thing," "S," is identical with two "things," "S-at-t_1" and "S-at-t_2". When I say that $e=mc^2$, I assert that "e" and "mc^2" are two "things" that are in reality one.

On the Divided Line, the objects of διάνοια are, basically, the propositions of mathematics, including definitions, axioms, and theorems. To shift from an arithmetic to a geometric example, the mathematician shows that a tangent touches a circle at only one point, using the definitions of tangent, circle, line, and point. The necessary truth expressed in the demonstration is a necessary truth about Being, the articulatable array of intelligible Forms that are unified by the One. According to the way Socrates describes the Line, διάνοια differs from νόησις because one who has the former does not see that a mathematical proposition is necessarily true because of its truth-maker, the One. The One in the guise of Good, as we learn explicitly, provides truth to the Forms and makes them knowable.[11] The truth it provides is ontological truth, the property possessed by Being itself of being transparent to an intellect. The semantic necessary truth of mathematical propositions is an expression of that. The knowability of Forms is, presumably, what makes it possible for the Demiurge to know Being, and for us somehow to know Being as well, specifically when we intellectually see that which unifies all intelligible being. In διάνοια we do not have knowledge. We can see the necessary truth of a mathematical proposition, but seeing that 3 + 4 = 7 is not seeing that the Form of Three and the Form of Four are relatively identical with the Form of Seven. This difference is the truth underlying Aristotle's claim that Plato posited mathematicals. When we add, we are not cognizing Forms since Forms are not addable. We are adding representations or manifestations of Forms. When we add (correctly) 3 and 4 and see that they are necessarily equal to 7, we do something that depends on their relative identity, but we do not cognize this. A contemporary philosopher of mathematics who happens to be a Platonist will say that 3 + 4 = 7 is an eternal and necessary truth, but will deny that this truth needs a truth-maker. She will deny that 3 and 4, on the one hand, and 7 on the other are identical in any way. The assertion of the brute facts of mathematics is no doubt why many philosophers of mathematics are disinclined to be Platonists because Plato insists that this is not the end of the story. Indeed, it is only the beginning of the story if you aim to understand reality comprehensively.

All cognition for Plato is a unificatory process. Cognition is strictly parallel to the unificatory process of reduction of Forms to Numbers and Numbers to the principles of One and Indefinite Dyad and, conversely, the derivation of all plurality from the One. I should add that, for Plato, moral progress is a unificatory process, too, and moral regress a process of dis-unification or dispersal. So, the virtuous person is one who becomes one out of many and the vicious person is one who is dis-unified by self-identification with a multitude of adventitious appetites. Our moral progress, as we learn in *Symposium*, *Republic*, and elsewhere, is primarily identification with our intellects. Thus, the unificatory process of cognition is continuous with the unificatory process of Platonic salvation. "Assimilation to god," or more accurately the process of making one the same as god requires moral virtue but is only achievable by philosophy, exactly as that is portrayed in the central books of *Republic*. The knowledge that is the mode of cognition of philosophers alone — given that the objects of this mode of cognition are recognized as existing only by philosophers — consists in seeing Being as a one-many. And this seeing is just becoming that one-many, that is, achieving cognitive identity with Being.

We may approach the metaphysical and epistemological vision developed in the Divided Line in another way. As we know from *Parmenides* (132B2ff), Plato makes a sharp distinction between a Form, which is intelligible (νοητόν) and a "concept" (νόημα). The concept is the Form as conceived by us. In διάνοια, the mathematician trades in concepts, for example, the concept of a plane figure or the concept of an odd number. These are λόγοι of the Forms. As Plato says in *Sophist*, διάνοια is just λόγος in the soul, that is, talking to oneself (263E3-5). When the διάνοια comes to a completion, so to speak, there is assertion (φάσις) and denial (κατάφασις) (E12). So, there is propositional content in διάνοια. Plato adds, by the way, that the name for this completion, whether it be an assertion or a denial is δόξα, a clear development of the scope of this term from *Republic*. Nevertheless, δόξα, even with this expansion of meaning, and ἐπιστήμη, are still sharply distinguished by their objects (cf. 533C8-D9).

The fact that one who has or pursues διάνοια is bound to use images seems to be one crucial point in the differentiation of διάνοια and ἐπιστήμη. The question that arises with regard to these images concerns their relation to the Forms themselves. How, for example, is the image of the circle used by the mathematician related to Circularity? This is of course a large and portentous topic, but here I want to distinguish between the instrumental use of the image and the content of the thinking. We won't get very far if we concentrate on a supposed two-term relation between, say, a picture of a circle and the Form of Circularity. We need, rather, a three-term relation which includes both the picture and the Form, but also includes the manner of cognition, that is, the manner of cognizing the νόημα. To think about circularity using the picture is to think the Form of Circularity universally. A Form itself is not a universal; nor is it a particular. The identical Form is present particularly in its instances and universally when it is cognized by us. In itself, it is neither universal nor particular. To cognize a Form universally is always to cognize it by means of an image of some sort. Perhaps paradoxically, thinking of Forms universally necessarily involves images, thus preventing our unqualified cognitive identity with these Forms. Trading in images, one cannot attain to ἐπιστήμη. The necessity and eternality that the mathematician sees in the truths of mathematics cannot be cogni-

tion of the internal relatedness of the Forms themselves. Saying that it is impossible that 3 + 4 equals anything but 7, is not seeing the relative identity of both sides of the equation. Rather, it is seeing an image of that internal relatedness. Cognizing universally is actually an impediment to the highest type of cognition. The Demiurge certainly does not have images of Forms; rather, he is cognizing the Forms themselves. He is cognitively identical with the Forms themselves. The Middle Platonic and then later, Christian, idea that the Forms are νόηματα in the mind of God misses this point. Aristotle is prepared to call ἐπιστήμη just this διάνοια, yet reserving a higher mode of cognition for the Unmoved Mover, namely, νόησις.

Of course, there are mathematicals, one sort of image of Forms. But there also seem to be non-mathematical or qualitative images of Forms. For example, to think that crimson is darker than pink is to think a necessary and eternal truth. Is the truth-maker for this truth an internal relation between Form-Numbers? It would seem so if the *only* way that the Demiurge puts intelligibility into the cosmos is by the use of shapes and numbers (εἴδεσί τε καὶ ἀριθμοῖς) (*Tim.* 53B1-5; cf. *Phil.* 24E7-25B2). The idea here is that intelligibility is mathematical in the sense of structural — whether dynamic or static — whereas the quasi-intelligible three-dimensional sensible world is compromised by spatiality or materiality, a principle of non-intelligibility manifested by the Receptacle. Owing to this principle, sensibles have scattered being (σκεδαστὴ οὐσία). The mathematical ordering of intelligible Being — what Descartes sought in a *mathesis universalis* — is imaged by a world that looks exactly like the one we live in. But the διάνοια that mathematicians engage in would not be possible if that mathematical ordering were not itself an image or expression of the first principle of all.

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NOTES

1 The fundamental divide is whether there are distinct objects of διάνοια, the so-called mathematical or whether these are Forms. Holding the former view are Adam (1920, 2nd edition 1963); Burnyeat (2000); Denyer (2007), among many others; holding the latter view with variations are Ross (1951); Murphy (1951); Smith (1996), among many others.

2 *Rep.* 507B9-10.

3 *Rep.* 509D6ff.

4 See *Meta.* A 6, 987b14-20, M 2, 1077a14ff.

5 See *Rep.* 511B2-D5 where νόησις and διάνοια are clearly distinguished by the fact that the first "attains" the Good whereas the second does not.

6 See *Eu*. 9C-11A.

7 For a lemmatized list of various interpretations of the Good and their principal supporters see Fronterotta (2001): 137-8, n.38.

8 See for a concise and comprehensive examination of the relevant indirect tradition Richard (2005), building on the seminal work of Krämer (1958, 2nd edition 1967), Gaiser (1963, 2nd edition 1968) and Szlezák (1988, 2nd edition 1992).

9 See *Parm*. 142C-145A.

10 See e.g, A 2, 185b32-34.

11 See *Rep*. 508E1-4 with 508A9-B7, 509B6, 17C2-3.